

# Game Mechanisms & Procedural Fairness

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The main paper to which this abstract refers models procedural fairness and procedural justice, crucial concepts in the design and appraisal of social interaction [1]. Argumentation systems in particular owe the quality of their inferences to the quality of the procedure that produces them. We are interested in both games that can be justified without reference to substantive social purposes, such as tournaments, and games that are fair, even if they are formally asymmetric, because they are appropriate to their social purposes, such as litigation and prosecution games.

A mathematical theory of procedural fairness has eluded theorists (much informal discussion in this regard has centered around the work of Rawls [2]). Such a theory would give formal standing to devices such as anonymization through chance and turn-swapping, equality of opportunity, symmetry of rules, and exchangeable asymmetries. It would understand the fundamental role of fair procedure: to construct justifiable ex post asymmetries that could not be justified except by reference to the procedure that produced them. Such a procedure begins with a justifiable ex ante position, and constructs its outcome on serendipity of play and chance, under an independently justifiable regimen.

We have developed a preliminary formal framework for exploring some mathematical properties of procedural fairness. The framework encompasses deterministic and stochastic games and player strategies, and contains formal devices for recursively composing complex games from simple components, including devices concerned with modifying/introducing procedural fairness properties. We also define allocation games as a special case, and may then invoke familiar game-theoretic concepts. Consider

$$j k p^+(m_0, m_1 \in \{Jan, Ken, Po\}) = \begin{cases} j k p^+ & \text{if } m_0 = m_1 \\ 0 & \text{if } (m_0, m_1) \in wins \\ 1 & \text{otherwise,} \end{cases} \quad (1)$$

a partial definition of *Jan-Ken-Po* (Rock-Paper-Scissors).

Chance mechanisms, in the form stochastic outcomes or symmetrical moves, may substitute for arbitrary choices and introduce fairness. For example, one may define, with our framework, a generic device for transforming a deterministic, asymmetrical game (such as chess), into a stochastic, symmetrical game (i.e., by flipping a coin to determine the assignment of player roles). Chance may also be used as a progress mechanism for states with indeterminable outcomes (e.g., re-

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solving deadlock by killing a random process). Of course, there are also situations where chance is undesirable because it creates unrepresentative outcomes. Often, this may be eliminated by *repetition*. For allocation games, this is best understood by looking at the spread of outcomes, which typically becomes “smoother” as a game is repeated.

Consider procedures designed to establish some social predicate such as guilt. It is constructive to distinguish between two kinds of fairness: *teleological fairness* (appropriateness, effectiveness) is concerned with the correspondence between what is measured by a procedure and what we intend it to; in contrast, *structural fairness* is concerned with internal aspects of procedures, such as arbitrariness or imbalance between players (if an even contest is intended). Time and spread are relevant to teleology; while repetition can decrease the risk of unrepresentative outcomes, it may introduce a wider spread, while devices such as “best of  $n$ ” games are time-consuming, and can limit perceived decisiveness.

In addressing physical or mental player limitations, there may be trade-offs between the conflicting goals of teleological and structural fairness. For example, in a “best of  $n$  games” match, increasing  $n$  improves structural fairness, while teleological fairness is decreased, as the match eventually degenerates into a measure of stamina rather than skill. Additionally, player limitations have a decisive impact of the teleological fairness or lack thereof of a given game. While tic-tac-toe is teleologically unfair for adults, it may be appropriate for young children.

Within our framework, *dominant strategies* may be defined absolutely, with respect to a particular population of strategies, or within a particular population of strategies. Tic-tac-toe has a dominant strategy, for example, but not within the population of strategies accessible to very young children.

Another important tool for reasoning about the procedural fairness properties of games is *reducibility*, which may be formally defined in terms of mappings between games. Interestingly, many actual games that at first glance appear reducible are not; reducibility depends on the population of players. For example, actual rock-paper-scissors matches may exhibit uncharacteristically long runs of ties, and are hence irreducible along simple lines. Note that the passage of time may be ignored by some reductions, while it in fact is the purpose of some actual games.

In conclusion, our preliminary work has hinted at a framework in which procedural devices may be isolated, and fairness claims can be derived. Much work remains to be done. Thus far, we have identified structural properties of chance, termination, spread, and rule symmetry, and commented on some of the relations between them in the appraisal of procedural fairness. As society is essentially built upon a foundation of procedural fairness, and a mathematical framework permits critical appraisal, we view this work to be of value.

## References

- [1] Michael D. Bayles. *Procedural Justice: Allocating to Individuals*. Kluwer Academic, New York, NY, 1990.
- [2] J. Rawls. *A Theory of Justice*. Belknap Press, Cambridge, MA, 1999.